Calculus & Vectors, Grade 12 (MCV4U)  
Course Description

Course Title: Calculus & Vectors  
Course Code: MCV4U  
Grade: 12

Course Type: University  
Credit Value: 1.0  
Prerequisite: MCR3U  
Co-requisite: MHF4U

• **This course builds on** your knowledge from grade 11 University mathematics and MHF4U  
• **It leads you** on a direct path to University and may be taken concurrently with MHF4U  
• **This can lead you to many careers such as:** Architect, Bio-Medical Engineer, General Manager (Sports Team), Dentistry


This course has three main strands:

Rate of Change

Derivatives and Their Applications

Geometry and Algebra of Vectors
Calculus & Vectors, Grade 12 (MCV4U)
Course Description

Rate of change:
Calculus is the study of rate of change. Students will learn about the limits of functions, explore average and instantaneous rates of change. For example: Determine the formula to calculate the average velocity between \(t_1\) and \(t_2\) and state how to determine the instantaneous velocity at \(t_1\). They will also learn the rules of differentiation and their applications to different types of functions (polynomial, exponential and sinusoidal).

**x-t Graph**

Average Velocity

Instantaneous Velocity

Average velocity is the slope of the line between two points
Instantaneous velocity is the slope of the tangent line at a specific point
Calculus & Vectors, Grade 12 (MCV4U)
Course Description

Derivatives and their applications:

Students will continue to use the rules of differentiation to solve real-world applications, such as calculating speed and distance using polynomial equations, radioactive decay problems, and price and inflation rate problems.

Problem: In order to get ready for the winter a city is dumping sand off a conveyor belt into a pile at a rate of 2 cubic metres per minute. The sand falls into a pile that is shaped like a cone with its height and base diameter always being equal. Safety regulations state that at the moment the pile reaches 5 m tall its height must not be increasing 2/5π metres per minute or the volume coming off the conveyor belt per minute must be reduced. Has the city set up a safe sand stockpiling?

Solution:

\[
\frac{dh}{dt} \bigg|_{h=5} = \frac{\pi}{4} \cdot \frac{h^2}{h} = \frac{\pi}{4} \cdot \frac{5^2}{5} = \frac{25\pi}{20} = 1.25 \text{ m/min}
\]

\[
\frac{dv}{dt} = \frac{\pi}{4} \cdot h^2 \cdot \frac{dh}{dt} = \frac{\pi}{4} \cdot 5^2 \cdot 1.25 = \frac{78.125\pi}{4} = 19.5625\pi \text{ m}^3/\text{min}
\]

\[
\frac{dv}{dt} = \frac{\pi}{4} \cdot h^2 \cdot \frac{dh}{dt}
\]

\[
\frac{dh}{dt} = \frac{dv}{dt} \bigg|_{h=5} = \frac{\pi}{4} \cdot 5^2 \cdot \frac{1}{2} = \frac{125\pi}{8} = 15.625\pi \text{ m/min}
\]

\[
\frac{dv}{dt} = \frac{\pi}{4} \cdot 5^2 \cdot 1.25 = \frac{78.125\pi}{4} = 19.5625\pi \text{ m}^3/\text{min}
\]

\[
\frac{25\pi}{20} = 1.25 \text{ m/min}
\]

\[
\frac{1}{2} \cdot \frac{25\pi}{20} = 0.25 \text{ m/min}
\]

\[
\frac{19.5625\pi}{0.25} = 80\pi \text{ m/min}
\]

\[
\frac{78.125\pi}{0.25} = 312.5\pi \text{ m}^3/\text{min}
\]

∴ This is a safe set up.
Calculus & Vectors, Grade 12 (MCV4U)  
Course Description

Geometry and algebra of vectors

Students will study lines and planes in both 2-space and 3-space, including the intersection of lines and planes. Students will also learn to add and subtract vectors in 2-space and 3-space. They will explore other properties and applications of vectors to solve problems like this:

**Problem:** If an airplane travels 335 km [N 30° E], then changes directions and travels 215 km [S 15° E], what was the airplane’s total displacement?

**Solution:**

Use the cosine law to solve for $x$

\[
x^2 = 335^2 + 215^2 - 2(335)(215)\cos 45°
\]

\[
x = \sqrt{335^2 + 215^2 - 2(335)(215)\cos 45°}
\]

\[
x = 237.9
\]

Angle of total displacement is

\[30° + 39.7° = 69.7°\]

∴ the airplane’s total displacement is

\[237.9 \text{ km } [N 69.7° E]\]

Use the sine law to solve for $\theta$

\[
\frac{\sin \theta}{215} = \frac{\sin 45}{237.9}
\]

\[
\theta = \sin^{-1}\left(\frac{215(\sin 45)}{237.9}\right)
\]

\[
\theta = 39.7°
\]
**Problem:** An interior designer is hanging a large decoration from the ceiling. The system looks like this diagram and is in equilibrium. She has a choice of 3 different wires to hang the decoration with: one can withstand tensions up to 50 N, the second up to 75N and the third can withstand up to 100N. The drawback is that the more tension it can withstand the thicker the wire which takes away from the overall aesthetics so she wants to use the thinnest wire possible. Which wire should she choose?

**Solution:**

\[ T_3 = 100 \text{ N} \]

\[ R_x = T_3 \cos 30^\circ - T_1 \cos 50^\circ = 0 \]

\[ T_2 = \frac{T_1 \cos 50^\circ}{\cos 30^\circ} \]

\[ R_y = T_1 \sin 50^\circ + T_2 \sin 30^\circ - 100 = 0 \]

\[ T_1 \sin 50^\circ + \frac{T_1 \cos 50^\circ}{\cos 30^\circ} \sin 30^\circ - 100 = 1 \]

\[ T_1 = \frac{100}{\sin 50^\circ + \frac{\cos 50^\circ}{\cos 30^\circ} \sin 30^\circ} = 87.94 \text{ N} \]

\[ T_2 = \frac{T_1 \cos 50^\circ}{\cos 30^\circ} = \frac{87.94 \cos 50^\circ}{\cos 30^\circ} = \frac{87.94 \times 0.643}{0.866} = 65.27 \text{ N} \]

\[ \therefore \text{ for string 1 she must use the thickest wire, but a thinner wire can be used for the other string.} \]