Calculus & Vectors, Grade 12 (MCV4U)
Course Description

Course Title: Calculus & Vectors
Course Code: MCV4U
Grade: 12

Course Type: University
Credit Value: 1.0
Prerequisite: MCR3U
Co-requisite: MHF4U

- **This course builds on** your knowledge from grade 11 University mathematics and MHF4U
- **It leads you** on a direct path to Universit and may be taken concurrently with MHF4U
- **This can lead you to many careers such as:** Architect, Bio-Medical Engineer, General Manager (Sports Team), Dentistry

Official Ontario Ministry of Education secondary curriculum available here:
http://www.edu.gov.on.ca/eng/curriculum/secondary/math.html

This course has three main strands:

Rate of Change

Derivatives and Their Applications

Geometry and Algebra of Vectors

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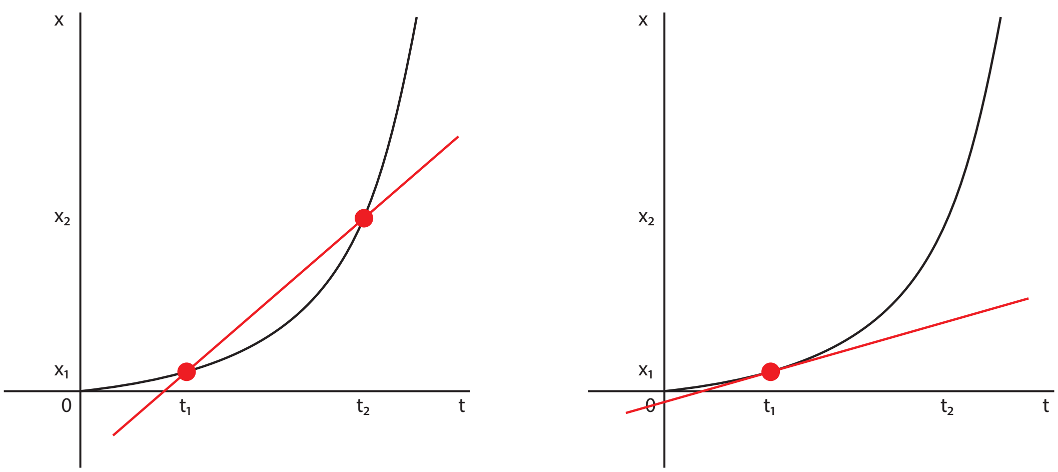
Rate of change:

Calculus is the study of rate of change. Students will learn about the limits of functions, explore average and instantaneous rates of change. For example: Determine the formula to calculate the average velocity between t_1 and t_2 and state how to determine the instantaneous velocity at t_1. They will also learn the rules of differentiation and their applications to different types of functions (polynomial, exponential and sinusoidal).

x-t Graph

Average Velocity | Instantaneous Velocity



Average velocity is the slope of the line between two points

Instantaneous velocity is the slope of the tangent line at a specific point

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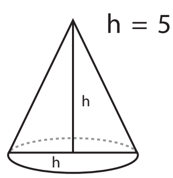
Derivatives and their applications:

Students will continue to use the rules of differentiation to solve real-world applications, such as calculating speed and distance using polynomial equations, radioactive decay problems, and price and inflation rate problems.

Problem: In order to get ready for the winter a city is dumping sand off a conveyor belt into a pile at a rate of 2 cubic metres per minute. The sand falls into a pile that is shaped like a cone with its height and base diameter always being equal. Safety regulations state that at the moment the pile reaches 5 m tall its height must not be increasing 2/5π metres per minute or the volume coming off the conveyor belt per minute must be reduced. Has the city set up a safe sand stockpiling?

Solution:



$$\left.\frac{dh}{dt}\right|_{h=5} = ?$$

$$\frac{dv}{dt} = 2\,\frac{m^3}{min}$$

$$v = \frac{\pi}{3} r^2 h$$

$$= \frac{\pi}{3}\left(\frac{1}{2}h\right)^2 h$$

$$= \frac{\pi}{12} h^3$$

$$\frac{d}{dt}(v) = \frac{d}{dt}\left(\frac{\pi}{12} h^3\right)$$

$$\frac{dv}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \left.\frac{\frac{dv}{dt}}{\frac{\pi}{4} h^2}\right|_{h=5}$$

$$= \frac{2\,\frac{m^3}{min}}{\frac{\pi}{4}\,0.25\,m^2}$$

$$= \frac{8}{25\pi}\ (m/min)$$

∴ This is a safe set up.

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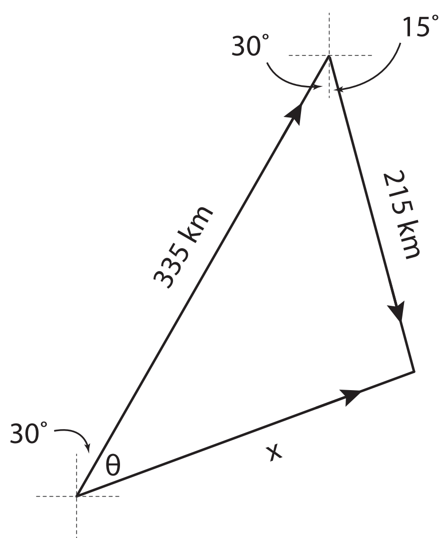
Geometry and algebra of vectors

Students will study lines and planes in both 2-space and 3-space, including the intersection of lines and planes. Students will also learn to add and subtract vectors in 2-space and 3-space. They will explore other properties and applications of vectors to solve problems like this:

Problem: If an airplane travels 335 km [N 30° E], then changes directions and travels 215 km [S 15° E], what was the airplane's total displacement?

Solution:



Use the cosine law to solve for x

$$x^2 = 335^2 + 215^2 - 2(335)(215)\cos 45^\circ$$

$$x = \sqrt{335^2 + 215^2 - 2(335)(215)\cos 45^\circ}$$

$$x = 237.9$$

Use the sine law to solve for θ

$$\frac{\sin\theta}{215} = \frac{\sin 45}{237.9}$$

$$\theta = \sin^{-1}\left(\frac{215(\sin 45)}{237.9}\right)$$

$$\theta = 39.7^\circ$$

Angle of total displacement is
$30^\circ + 39.7^\circ = 69.7^\circ$

∴ the airplanes total displacement is
237.9 km [N 69.7° E]

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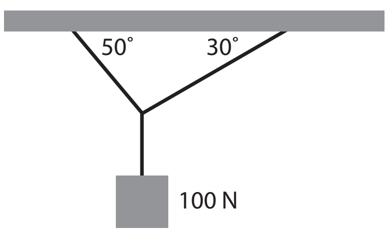
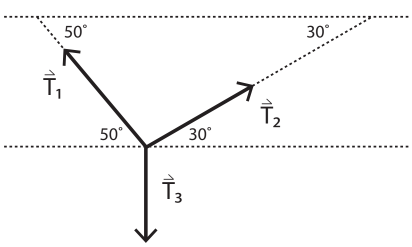
Problem: An interior designer is hanging a large decoration from the ceiling. The system looks like this diagram and is in equilibrium. She has a choice of 3 different wires to hang the decoration with: one can withstand tensions up to 50 N, the second up to 75N and the third can withstand up to 100N. The drawback is that the more tension it can withstand the thicker the wire which takes away from the overall aesthetics so she wants to use the thinnest wire possible. Which wire should she choose?

Solution:





$T_3 = 100\text{ N}$

$R_x = T_2 \cos 30^\circ - T_1 \cos 50^\circ = 0$

$$T_2 = \frac{T_1 \cos 50^\circ}{\cos 30^\circ}$$

$R_y = T_1 \sin 50^\circ + T_2 \sin 30^\circ - 100 = 0$

$$T_1 \sin 50^\circ + \frac{T_1 \cos 50^\circ}{\cos 30^\circ} \sin 30^\circ - 100 = 1$$

$$T_1 = \frac{100}{\sin 50^\circ + \frac{\cos 50^\circ}{\cos 30^\circ} \sin 30^\circ}$$

$$= 87.94\text{ N}$$

$$T_2 = \frac{T_1 \cos 50^\circ}{\cos 30^\circ}$$

$$= \frac{87.94 \cos 50^\circ}{\cos 30^\circ}$$

$$= \frac{87.94 \times 0.643}{0.866}$$

$$= 65.27\text{ N}$$

∴ for string 1 she must use the thickest wire, but a thinner wire can be used for the other string.

Trigonometric Table

Angle in degrees	Angle in Radians	Sine	Cosine	Tangent	Angle in degrees	Angle in Radians	Sine	Cosine	Tangent
26˚	0.454	0.438	0.899	0.488	41˚	0.716	0.656	0.755	0.869
27˚	0.471	0.454	0.891	0.510	42˚	0.733	0.669	0.743	0.900
28˚	0.489	0.469	0.883	0.532	43˚	0.750	0.682	0.731	0.933
29˚	0.506	0.485	0.875	0.554	44˚	0.768	0.695	0.719	0.966
30˚	0.524	0.500	0.866	0.577	45˚	0.785	0.707	0.707	1.000
31˚	0.541	0.515	0.857	0.601	46˚	0.716	0.656	0.695	1.036
32˚	0.559	0.530	0.848	0.625	47˚	0.733	0.669	0.682	1.072
33˚	0.576	0.545	0.839	0.649	48˚	0.750	0.682	0.669	1.111
34˚	0.593	0.559	0.829	0.675	49˚	0.768	0.695	0.656	1.150
35˚	0.611	0.574	0.819	0.700	50˚	0.785	0.707	0.643	1.192